Determination of the Earth-Sun distance
Mercury transit: 11/11/2019

The principle in itself is not very complex: seen from two observers located at a great distance from each other, on Earth, Mercury photographed at the same time in front of the Sun, must appear to be located in projection, by parallax , at two different locations on the solar disk.

The difference between the two perceived positions depends on the Earth-Sun distance and the Mercury-Sun distance.

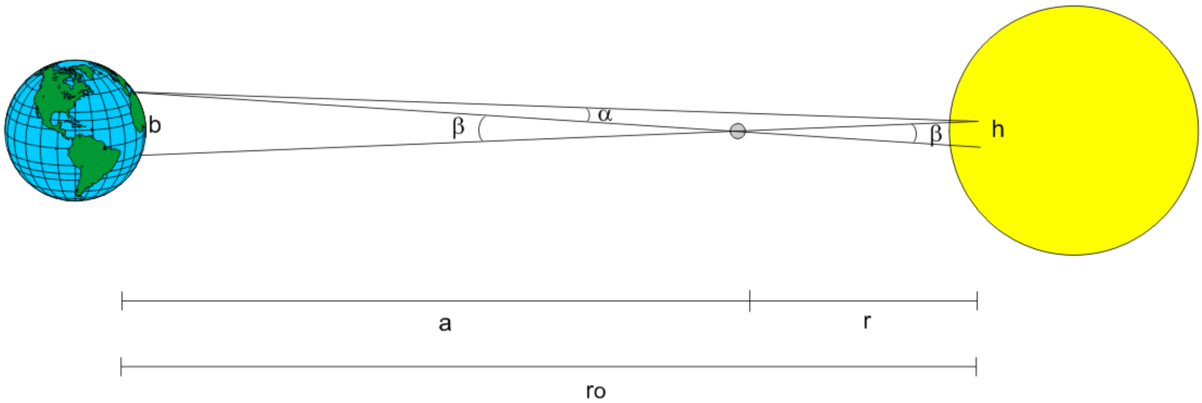
A partnership has thus been set up between the Astro ERATO Lab at the Lycée Jacques Ruffié and a club of amateur astronomers in Pretoria (ASSA), in South Africa.

In practice, the two difficulties are to be able to determine:

1/ the positional shift of Mercury between the two images

2/ The distance between the two observers, projected on a line perpendicular to the Sun-Earth line.

A little theory:



α: is the angular offset between the two images of Mercury
b: is the distance between the two observers, projected on a line of sight perpendicular to the Earth-Sun line
ro: what is sought, the Earth-Sun distance

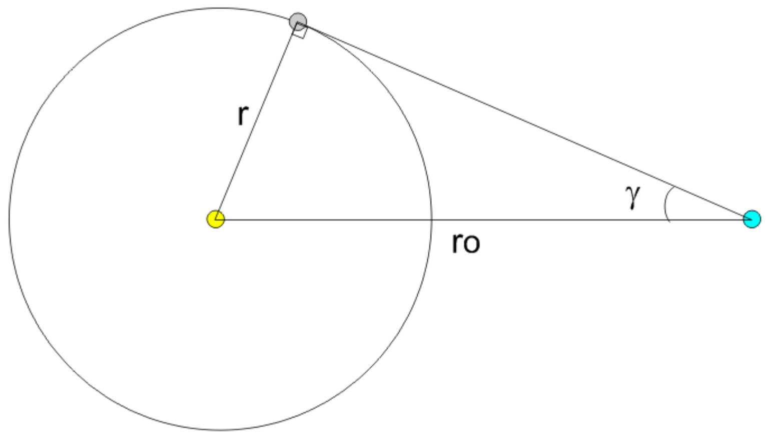
$$\alpha = \frac{h}{ro}$$

$$\beta = \frac{b}{a} = \frac{h}{r} = \frac{b}{ro - r} = \frac{\alpha . ro}{r}$$

$$\frac{b}{\alpha} = \frac{ro.(ro - r)}{r} = ro.(\frac{ro}{r} - 1)$$

b and α being measured, it suffices to take **ro** out of the equation, but **r** is also unknown!...

An additional observation is then necessary, the maximum elongation (γ) of Mercury:



According to this diagram,
sin γ = r/ro

The previous formula then becomes:

$$\frac{b}{\alpha} = ro.\left(\frac{1}{\sin\gamma} - 1\right)$$

where we come from:

$$ro = \frac{b/\alpha}{1/\sin\gamma - 1}$$

Experimental determinations:

Two observation sites are in place:

Limoux in France and the students of the Astro ERATO Lab:

Pretoria in South Africa, with Johan Moolman (and thanks to Neville Young, Johan Smit, Percy Jacobs from ASSA and other South African contributors):

Observation protocol:

-The images are obtained in such a way that the Sun is fully present on the image

-The sensor is chosen to have sufficient resolution

-The observers agree to make an image every 10 min full throughout the transit, which lasts all afternoon.

Determination of the offset between the two Mercures: problem of image alignment.

-In Limoux, the images are taken with a reflex camera (Canon EOS 6D) in the foyer of the C8 loaned by the Meudon observatory (Astro project at school). The mounting of the instrument is equatorial, so, from one image to another the orientation of the photographed field remains the same.

-In Pretoria, Johan communicates to us first of all very beautiful images, but his mount was azimuthal: a rotation of field thus appears from one image to another, they are therefore not usable.

A second series is provided to us by Johan, very efficient, obtained with an equatorial mount. These are usable!

BUT: we were hoping for some sunspots, which would have made it possible to align the images between Pretoria and Limoux. In bad luck, the Sun, in minimum activity, is uniform, with no reference point. Two images cannot therefore be aligned.

A SOLUTION: If we use not two images taken at the same time from two places, but rather ALL the images taken at the two places, we are left with two movements of Mercury on the Sun, from France or from South Africa .

Here is for example the version obtained in France:

So, now, with a little patience and delicacy, the two trajectories of Mercury become alignable, of course after scaling.

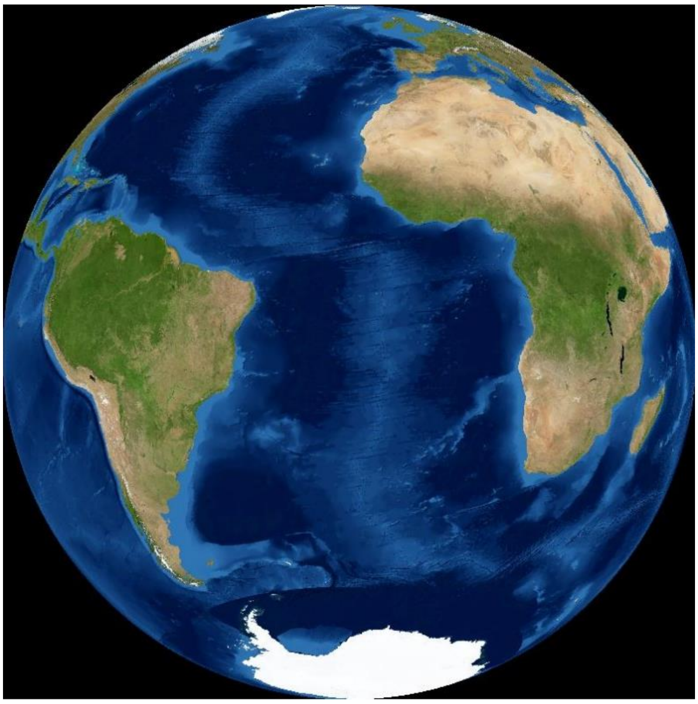
The black dots are those of the South, the whites are those of the North.

The pupils measure, by comparison with the apparent diameter of the Sun that day, **an offset of 6.5 seconds of angle on average, that's α.**

Maximum elongation of Mercury:

Difficult weather conditions lead us to use astronomical simulation software (SkyChart from Patrick Chevalley), which leads to a determination of **γ = 23 °**

Distance between the two observers:



This measurement, performed on the line perpendicular to the Sun-Mercury line, is complex to determine by calculating astronomical positions. A trick simplifying the problem was used:

The FOURMILAB website allows you to obtain simulated images of objects in the solar system seen from others, at selected dates and times.

It thus makes it possible to obtain this image of the Earth, as being a view taken from the Sun, at the time of transit.

A simple proportionality based on the real diameter of the Earth and the **distance measured on the image between Limoux and Pretoria allows to obtain b = 7400 km**

Final determination of the Earth-Sun distance:

All the values obtained above are combined, in the right units, thanks to the formulas previously established, the final result obtained is:

151 million km

That day, the ephemeris predicted 149 millions km

Or about 1% deviation from the theoretical value ...

PhB for the ERATO Lab